Tidal deformation of black holes and neutron stars

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Tidal deformations of neutron stars affect their orbital motion, and this can have a measurable impact on the emitted gravitational waves [Flanagan and Hinderer (2008)]

\[ M_{NS} = 1.4 M_\odot = M_{BH} / 3 \]

Gravitational-wave measurement of inspirals can reveal information about the internal structure of neutron stars [Pannarale, Rezzolla, Ohme, Read (2011)]
Newtonian theory of tides

Tidal forces are produced by inhomogeneities in the gravitational field produced by a remote body.

\[ U_{\text{ext}} = U_{\text{ext}}(0) + g_a x^a - \frac{1}{2} \mathcal{E}_{ab} x^a x^b + \cdots \]

\[ Q_{ab} = \int \rho (x_a x_b - \frac{1}{3} r^2 \delta_{ab}) dV \]

\[ Q_{ab} = -\frac{2}{3} k_2 R^5 \mathcal{E}_{ab} \]

The Love number \( k_2 \) encapsulates the details of the body’s internal structure.
The physics external to the body involves the gravitational field only
\[ \nabla^2 U = 0 \]

- Growing solution (tidal field): \( U \sim \mathcal{E} r^2 \)
- Decaying solution (tidal response): \( U \sim k_2 / r^3 \)
- The precise mixture of growing and decaying solutions is determined by matching with the internal solution
- This determines the **Love number** \( k_2 \)
Internal physics

- The physics inside the body involves gravity and the matter variables.
- A simple matter model is a perfect fluid (density, pressure, velocity).
- The effect of the tidal forces can be determined in perturbation theory.
- All variables can be decomposed in normal modes, and each mode behaves as a simple harmonic oscillator.

\[ \ddot{\xi} + \omega^2 \xi = f \]

- External tidal force
- Restoring force (pressure gradients)
- For a slowly changing tidal field: \( \xi = f/\omega^2 \) (perturbed equilibrium)
Tides in GR

* There are two types of tidal fields in general relativity: gravitoelectric and gravitomagnetic.

* Given a timelike vector field, the Weyl tensor can be decomposed as

\[ \mathcal{E}_{\alpha\beta} = u^\mu u^\nu C_{\alpha\mu\beta\nu} \]

\[ \mathcal{B}_{\alpha\beta} = \frac{1}{2} u^\mu u^\nu \epsilon_{\mu\alpha\gamma\delta} C^{\gamma\delta}_{\beta\nu} \]

* Much of post-Newtonian gravity is captured by scalar and vector potentials,

\[ \nabla^2 U = -4\pi \rho \quad \nabla^2 U_\alpha = -4\pi \rho u_\alpha \]
Nonrotating bodies

- **External physics:** perturbed Schwarzschild metric
  - Growing solution: regular at $r=2M$, diverges at infinity
  - Decaying solution: diverges at $r=2M$, vanishes at infinity
  - The precise mixture of growing and decaying solutions is determined by matching with the internal solution
  - This determines the **relativistic Love number** $k_2$

- **Internal physics:** perturbed fluid and internal metric
Relativistic Love number

\[ k_2 \]

\[ M/R = n \]

\[ n=2.0 \quad n=1.5 \quad n=1.0 \quad n=0.5 \quad n=0.0 \]

[SQM 1.0 - 3]

[Normal]

[Hinderer, Lackey, Lang, Read (2010)]
Relativistic Love number

- For a black hole

\[ k_2 = 0 \]

(because the decaying solution must be eliminated)

[Binnington and Poisson (2009)]
Rotating black hole

* The tidal deformation of a rotating black hole can be computed with the help of the Teukolsky equation [O’Sullivan and Hughes (2014, 2016); Penna, Hughes, O’Sullivan (2017)]

* When $\chi = J/M^2 \ll 1$ the rotational deformation of the black hole can be neglected, and the tidal deformation incorporates all effects associated with the dragging of internal frames [Poisson (2015)]

* The rotation produces a phase lag (not lead) between the horizon’s tidal bulge and the tidal field: $(\Omega_{\text{tide}} \ll \Omega_H)$

  $$\varphi_{\text{bulge}} = (\Omega_{\text{tide}} - \Omega_H) \nu - \frac{2}{3} \chi$$
Rotating neutron star

* The coupling between rotation and tidal field requires the introduction of new (octupolar) Love numbers

[Landry and Poisson (2015); Pani, Gualtieri, Ferrari (2015)]

\[
\delta g_{tt} \sim \frac{MR^5}{r^4} \chi^{(a} B_{bc)} n^a n^b n^c
\]

\[
\delta g_{ta} \sim \frac{M^5 R}{r^4} f^o \epsilon_{ab}^{\ c} \chi^{(c} E_{de)} n^b n^d n^e
\]

\[
n^a = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)
\]
\[ \text{Landry (2017)} \]
Gravitomagnetic tidal currents

* The coupling between the rotational velocity and the gravitomagnetic tidal field produces a force density within the neutron star

\[ f = \rho \nu \times B, \quad B = \nabla \times U \]

* This force is not balanced by pressure gradients

\[ \ddot{\xi} + \omega^2 \xi = f \]

* This leads to the development of a large velocity field

[Landry and Poisson (2015); Poisson and Doucot (2017); Landry (2017)]

\[ \delta \nu = 2 \left( \frac{M'}{1.4 M_\odot} \right) \left( \frac{2.8 M_\odot}{M + M'} \right)^{2/3} \left( \frac{R}{12 \text{ km}} \right)^2 \left( \frac{100 \text{ ms}}{P} \right) \left( \frac{f}{100 \text{ Hz}} \right)^{4/3} \text{ km/s} \]
Conclusion

- A compact object undergoes a tidal deformation as it orbits around another compact object.
- This affects the orbital motion and the emitted gravitational waves.
- This effect can be measured in gravitational waves during the inspiral phase of the binary motion.
- A measurement of the tidal Love number reveals information about the internal structure of the compact body.
- In the case of a rotating neutron star, the coupling between the rotation and the gravitomagnetic tidal fields generates large tidal currents.